Mutually Orthogonal Latin Squares

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There is a long standing open problem concerning the maximum number of mutually orthogonal Latin squares (MOLS) of side n. Let $N(n)$ denote the maximum number of MOLS of side n. It is known that $\lim_{n\to\infty} N(n) \geq n^{1/17} - 2$. Similarly, there is an open question on the maximum cardinality, denoted M(n,d), of a set of permutations on n symbols with Hamming distance d. The connection between $N(n)$ and $M(n, n-1)$ is: If there are k MOLS of side n, then $M(n, n-1) \geq kn$ [1]. Let $H(n)$ denote $M(n, n-1)/n$, so we have (by [1]) $\lim_{n\to\infty} H(n) \geq n^{1/17} - 2$. Presumably, this bound can be substantially improved.

Question 1 What is $\lim_{n\to\infty} H(n)$?

It has been shown that infinitely often, $H(n)$ is larger than known values of $N(n)$ [2]. This is done by partition and extension.

Question 2 Is $M(n, n-1) \geq (n-1) \cdot \min(\lfloor\sqrt{n-1}\rfloor, N(n-1))$, for all n?

Based on computational results, it is true for all but four cases of $n \leq 600$. We believe these computations can be improved and the inequality is true for all n.

[1] Colbourn, Love, Ling; IEEE Trans. Inf. Theory, 2004.
[2] Bereg, Morales, and Sudborough; Designs, Codes, and Cryptography, 2017.

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